SCALE CITY

KET

# The Road to Proportional Reasoning: Belle of Louisville Handouts

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# BELLE OF LOUISVILLE: HANDOUT 1

## "Greetings from the Belle of Louisville" Video Notes

**Name:** **Date:**

Watch and listen carefully to complete the blanks with the following terms:

- pitch
- 32
- lengths
- octave
- Belle
- 8
- 16
- longest
- shortest
- shortens

**1.** The ________________ of Louisville is a steam-powered paddlewheel boat, close to 100 years old, based in Louisville, Kentucky.

**2.** The Belle's calliope is an old-fashioned steam-powered instrument with ________________ pipes.

**3-4.** The deepest sound comes from the ________________ pipe. The highest pitch comes from the ________________ pipe. The largest pipe is over two feet tall with a diameter of seven inches.

**5.** The church organ built in 1929 has three thousand pipes that produce sounds proportionate to their size. The smallest pipe here measures a mere one inch, while the largest pipe reaches _______ feet.

**6.** The Greek philosopher Pythagoras is said to have first connected music and mathematics. Pythagoras discovered that the length of the string determined the ________________ and that musical notes are related to ratios.

**7.** A glockenspiel, like a xylophone, has bars instead of pipes. The different ________________ of the bars create different pitches.

**8.** Putting your finger on the fret of a bass guitar ________________ the string and produces a different pitch.

**9.** When a group of notes are arranged in ascending or descending order and span an ________________, it is a called a scale.

**10.** An octave is a musical interval of ________________ tones.

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# KEY: BELLE OF LOUISVILLE: HANDOUT 1

## "Greetings from the Belle of Louisville" Video Notes

**1.** The ***Belle*** of Louisville is a steam-powered paddlewheel boat, close to 100 years old, based in Louisville, Kentucky.

**2.** The Belle's calliope is an old-fashioned steam-powered instrument with ***32*** pipes.

**3-4.** The deepest sound comes from the ***longest*** pipe. The highest pitch comes from the ***shortest*** pipe. The largest pipe is over two feet tall with a diameter of seven inches.

**5.** The church organ built in 1929 has three thousand pipes that produce sounds proportionate to their size. The smallest pipe here measures a mere one inch, while the largest pipe reaches ***16*** feet.

**6.** The Greek philosopher Pythagoras is said to have first connected music and mathematics. Pythagoras discovered that the length of the string determined the ***pitch*** and that musical notes are related to ratios.

**7.** A glockenspiel, like a xylophone, has bars instead of pipes. The different ***lengths*** of the bars create different pitches.

**8.** Putting your finger on the fret of a bass guitar ***shortens*** the string and produces a different pitch.

**9.** When a group of notes are arranged in ascending or descending order and span an ***octave***, it is a called a scale.

**10.** An octave is a musical interval of ***8*** tones.

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# BELLE OF LOUISVILLE: HANDOUT 2

## Musical Math

**Name:** **Date:**

Complete the chart from the online interactive, "Musical Scales," at **www.scalecity.org**.

| Note | Frequency | Length | Frequency × Length* |
|---|---|---|---|
| C4 | | | |
| D4 | | | |
| E4 | | | |
| F4 | | | |
| G4 | | | |
| A4 | | | |
| B4 | | | |
| C5 | | | |

**Values are approximate and rounded. An actual pan flute of this size may produce different values. In the interactive, after you enter the product of frequency × length in the table and check answers, correct answers automatically round to 2350, the nearest 50.*

C4 is what piano players think of as middle C, the C in the middle of the full keyboard. C5 is one octave to the right, that is, one octave higher in pitch. C3 represents the C key an octave lower in pitch, on the left.

**1.** As the pitch gets higher, what happens to the frequency?

**2.** As the length of the pan flute pipe increases, what happens to the frequency? Observing the relationship of frequency and length, how would you describe this relationship?

**3.** What is the ratio of the frequency of C5 to the frequency of C4?

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**4.** C4 and C5 are an octave apart. Using your ratio from question 3 as the basis for your reasoning, determine what the frequency would be of the following:
A. C3

B. C6

C. C8

**5.** Much study has been done on the relationship of intervals between notes. This is often called harmonics.

A. In musical terms, C4 and G4 are a perfect fifth apart. Using the frequencies in the chart above, what is the value of the ratio of G4's frequency to C4's frequency? Round your answer to the nearest hundredth.

B. The ratio of a perfect fifth is 3:2 or 3/2. How does your calculation in Part A compare with this ratio?

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# KEY: BELLE OF LOUISVILLE: HANDOUT 2

## Musical Math

Complete the chart from the online interactive, "Musical Scales," at **www.scalecity.org**.

| Note | Frequency | Length | Frequency × Length* |
|---|---|---|---|
| C4 | *261.63* | *9* | *2350* |
| D4 | *293.66* | *7.99* | *2350* |
| E4 | *329.63* | *7.13* | *2350* |
| F4 | *349.23* | *6.74* | *2350* |
| G4 | *392.00* | *6.00* | *2350* |
| A4 | *440.00* | *5.35* | *2350* |
| B4 | *493.88* | *4.76* | *2350* |
| C5 | *523.26* | *4.5* | *2350* |

**Values are approximate and rounded. An actual pan flute of this size may produce different values. In the interactive, after you enter the product of frequency × length in the table and check answers, correct answers automatically round to 2350, the nearest 50.*

C4 is what piano players think of as middle C, the C in the middle of the full keyboard. C5 is one octave to the right, that is, one octave higher in pitch. C3 represents the C key an octave lower in pitch, on the left.

**1.** As the pitch gets higher, what happens to the frequency?
*The frequency is greater.*

**2.** As the length of the pan flute pipe increases, what happens to the frequency? Observing the relationship of frequency and length, how would you describe this relationship?
*The frequency gets smaller as the length of the pipe increases. This is an inverse proportion. Frequency × length produces a constant (k) of approximately 2350, again demonstrating that frequency and length are inversely proportional. As length increases, frequency decreases.*

**3.** What is the ratio of the frequency of C5 to the frequency of C4?
*523.26/261.63 = 2/1 or 2:1*

**4.** C4 and C5 are an octave apart. Using your ratio from question 3 as the basis for your reasoning, determine what the frequency would be of the following:
*A. C3  261.63/2 = 130.815*
*B. C6 523.26 × 2 = 1046.52*
*C. C8 1046.52 × 2 × 2 = 4186.08*

**5.** Much study has been done on the relationship of intervals between notes. This is often called harmonics.

A. In musical terms, C4 and G4 are a perfect fifth apart. Using the frequencies in the chart above, what is the value of the ratio of G4's frequency to C4's frequency? Round your answer to the nearest hundredth.
*G4/C4 = 392 ÷ 261.63 = 1.498 = 1.50*

B. The ratio of a perfect fifth is 3:2 or 3/2. How does your calculation in Part A compare with this ratio?
*Equal or a close approximation.  3/2 = 3 ÷ 2 =1.5. The value of the ratio of G4/C4 rounds to 1.5.*

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# BELLE OF LOUISVILLE: HANDOUT 3

## Tuning Time

**Name:** **Date:**

**1.** "Just intonation" is a system of musical tuning in which ratios of whole numbers describe the relationships between notes. These intervals are based on the ratio of frequencies of two pitches. Complete the decimal value of each ratio in a "just intonation" scale.

| Name of Relationship | Example | Ratio of Frequencies | Decimal Value |
|---|---|---|---|
| Perfect unison | C to C | 1:1 | 1.0 |
| Major second | D to C | 9:8 | |
| Major third | E to C | 5:4 | |
| Perfect fourth | F to C | 4:3 | |
| Perfect fifth | G to C | 3:2 | |
| Major sixth | A to C | 5:3 | |
| Major seventh | B to C | 15:8 | |
| Perfect octave | C octave to C | 2:1 | |

**2.** To make a pan flute an octave higher than the one at the "Musical Scales" interactive, use the information about just intonation to complete the chart. Hint: There are lots of ways to solve these problems using the ratio of frequencies, the decimal value of the ratios, and your knowledge of inverse proportions. And when you figure out the frequencies, that information can help you determine the length of the pipes.

| Note | Frequency | Length (in) | Frequency x Length |
|---|---|---|---|
| C5 | 523.26 | 4.5 | |
| D5 | | | |
| E5 | | | |
| F5 | | | |
| G5 | | | |
| A5 | | | |
| B5 | | | |
| C6 | | | |

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**3.** We have learned that a note an octave higher has twice the frequency of the same note an octave below. We have also learned that a pipe with half the length of another pipe produces a sound an octave higher. So it follows that an octave lower is half the frequency at twice the pipe length. Use this information to complete the chart below. Round the product of Frequency x Length to the nearest 50.

| Note | Frequency | Length (in) | Frequency x Length |
|---|---|---|---|
| C4 | 261.63 | 9 | 2350 |
| C3 | | | |
| D4 | 293.66 | 7.99 | 2350 |
| D3 | | | |
| E4 | 329.63 | 7.13 | 2350 |
| E5 | | | |
| F4 | 342.23 | 6.74 | 2350 |
| F5 | | | |
| G4 | 392.00 | 6.00 | 2350 |
| G5 | | | |
| A4 | 440.00 | 5.35 | 2350 |
| A5 | | | |
| B4 | 493.88 | 4.76 | 2350 |
| B3 | | | |
| C5 | 523.26 | 4.5 | 2350 |
| C4 | | | |

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# KEY: BELLE OF LOUISVILLE: HANDOUT 3

## Tuning Time

**1.** "Just intonation" is a system of musical tuning in which ratios of whole numbers describe the relationships between notes. These intervals are based on the ratio of frequencies of two pitches. Complete the decimal value of each ratio in a "just intonation" scale.

| Name of Relationship | Example | Ratio of Frequencies | Decimal Value |
|---|---|---|---|
| Perfect unison | C to C | 1:1 | 1.0 |
| Major second | D to C | 9:8 | *1.125* |
| Major third | E to C | 5:4 | *1.25* |
| Perfect fourth | F to C | 4:3 | *1.3333* |
| Perfect fifth | G to C | 3:2 | *1.5* |
| Major sixth | A to C | 5:3 | *1.6667* |
| Major seventh | B to C | 15:8 | *1.875* |
| Perfect octave | C octave to C | 2:1 | *2.0* |

**2.** To make a pan flute an octave higher than the one at the "Musical Scales" interactive, use the information about just intonation to complete the chart. Hint: There are lots of ways to solve these problems using the ratio of frequencies, the decimal value of the ratios, and your knowledge of inverse proportions. And when you figure out the frequencies, that information can help you determine the length of the pipes.

| Note | Frequency | Length | Frequency x Length |
|---|---|---|---|
| C5 | 523.26 | 4.5 | 2354.67 |
| D5 | *588.6675* | *4* | *2354.67* |
| E5 | *654.075* | *3.6* | *2354.67* |
| F5 | *697.68* | *3.375* | *2354.67* |
| G5 | *784.89* | *3.0* | *2354.67* |
| A5 | *872.1* | *2.7* | *2354.67* |
| B5 | *981.1125* | *2.4* | *2354.67* |
| C6 | *1046.52* | *2.25* | *2354.67* |

**Note to teachers:**

*The frequencies and lengths of the pipe are both calculated here using just intonation and proportional reasoning. Students might determine frequencies by multiplying the frequency for C5 by the decimal values in the table above. Or they might set up proportions as in the example below.*

*x = unknown frequency for D5*
*x/523.26 (frequency of C5) = 9/8*
*x = 9 • 523.26/8*
*x = 4709.34/8*
*x = 588.6675*

*If they know the frequencies and lengths for C4, D4, E4, etc., it is also easy to double the frequencies and lengths for C5, D5, E5, etc. and halve the lengths. So students could refer back to the table created in the interactive.*

*Here are several additional ways the students could determine the length of the pipes on their pan flute, using D5 as an example.*

*Since the length of the pipe is inversely proportional to its frequency, they could use what they know about inverse proportions (xy = k where x and y are two related variables and k is a constant) to solve for length. So the frequency of a note times its length should equal the constant 2354.67.*

$588.6675x = 2354.67$
$x = 4$

*Or they could set up an inverse proportion:*

*Frequency of C5/Frequency of D5 = Length of pipe D5/Length of pipe C5*
$523.26/588.66675 = x/4.5$

*Or they could divide the length of pipe C5 by the decimal value of the ratio of its frequency to the frequency of D5.*

**3.** We have learned that a note an octave higher has twice the frequency of the same note an octave below. We have also learned that a pipe with half the length of another pipe produces a sound an octave higher. So it follows that an octave lower is half the frequency at twice the pipe length. Use this information to complete the chart below. Round the product of Frequency x Length to the nearest 50.

| Note | Frequency | Length (in) | Frequency x Length |
|---|---|---|---|
| C4 | 261.63 | 9 | 2350 |
| C3 | *130.815* | *18* | *2350* |
| D4 | 293.66 | 7.99 | 2350 |
| D3 | *146.83* | *15.98* | *2350* |
| E4 | 329.63 | 7.13 | 2350 |
| E3 | *164.815* | *14.26* | *2350* |
| F4 | 349.23 | 6.74 | 2350 |
| F3 | *174.615* | *13.48* | *2350* |
| G4 | 392.00 | 6.00 | 2350 |
| G3 | *196.00* | *12.00* | *2350* |
| A4 | 440.00 | 5.35 | 2350 |
| A3 | *220.00* | *10.7* | *2350* |
| B4 | 493.88 | 4.76 | 2350 |
| B3 | *246.94* | *9.52* | *2350* |
| C5 | 523.26 | 4.5 | 2350 |
| C4 | *261.63* | *9* | *2350* |

# BELLE OF LOUISVILLE: HANDOUT 4

## What Is a Pythagorean Scale?

**Name:** **Date:**

It is said that Pythagoras was walking by a blacksmith shop when he noticed the different pitches that the hammers made. From there, he began experimenting with ratio and musical scale to understand the order of music.

Pythagoras himself played a stringed instrument called the lyre. He lived before the days when frequency was measured using Hertz. He observed that when one string was twice as long as another string, with a 2:1 ratio, the notes were harmonious and exactly one octave apart, and that the shorter the string, the higher the sound. Pythagoras is often credited with a method of tuning based on a 3:2 ratio. Notes that are five whole steps apart are said to have this 3:2 ratio.

The ratios of Pythagoras listed below describe an ascending scale. In terms of frequency, theoretically the note at 2:1 would have twice the frequency of the note at 1:1. The relationship of frequency and string length is an inverse proportion.

Use the ratios of Pythagoras and your knowledge of inverse proportions to specify the measurements of an instrument using vibrating string. Round your answers to the nearest hundredth. (Hint: Use the inverse of the Pythagorean ratios to determine the length of the strings. Your values should be between 16 and 8 inches.)

| Pythagoras Ratio to First Note of Scale | Length of String |
|---|---|
| 1/1 | 16 inches |
| 9/8 | |
| 81/64 | |
| 4/3 | |
| 3/2 | |
| 27/16 | |
| 243/128 | |
| 2/1 | 8 inches |

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# KEY: BELLE OF LOUISVILLE: HANDOUT 4

## What Is a Pythagorean Scale?

It is said that Pythagoras was walking by a blacksmith shop when he noticed the different pitches that the hammers made. From there, he began experimenting with ratio and musical scale to understand the order of music.

Pythagoras himself played a stringed instrument called the lyre. He lived before the days when frequency was measured using Hertz. He observed that when one string was twice as long as another string, with a 2:1 ratio, the notes were harmonious and exactly one octave apart, and that the shorter the string, the higher the sound. Pythagoras is often credited with a method of tuning based on a 3:2 ratio. Notes that are five whole steps apart are said to have this 3:2 ratio.

The ratios of Pythagoras listed below describe an ascending scale. In terms of frequency, theoretically the note at 2:1 would have twice the frequency of the note at 1:1. The relationship of frequency and string length is an inverse proportion.

Use the ratios of Pythagoras and your knowledge of inverse proportions to specify the measurements of an instrument using vibrating string. Round your answers to the nearest hundredth. (Hint: Use the inverse of the Pythagorean ratios to determine the length of the strings. Your values should be between 16 and 8 inches.)

| Pythagoras Ratio to First Note of Scale | Length of String |
|---|---|
| 1/1 | 16 inches |
| 9/8 | *14.22 inches* |
| 81/64 | *12.64 inches* |
| 4/3 | *12 inches* |
| 3/2 | *10.67 inches* |
| 27/16 | *9.48 inches* |
| 243/128 | *8.43 inches* |
| 2/1 | 8 inches |

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# BELLE OF LOUISVILLE: HANDOUT 5

## Enrichment: Fractions in Musical Notation

**Name:** **Date:**

The timing of music uses a special written notation. Written music begins with a time signature that has two numbers. Many songs are written in what is called 4/4 time or 3/4 time (pronounced "four four" and "three four"). The number on top (here 4 or 3) describes how many beats there are in a measure, the even time intervals into which written music is divided.

The bottom number describes which note receives one beat. In typical 4/4 time and 3/4 time, a quarter note gets one beat. The following notes are held for these durations with these time signatures.

Whole notes = 4 beats
1/2 notes = 2 beats
1/4 notes = 1 beat
1/8 notes = 1/2 beat
1/16 notes = 1/4 beat

**1.** For 3/4 time, choose a note or combination of notes that could complete a measure that starts with a half note.

**2.** In one measure of music in 4/4 time, eight 16th notes are used with a rest for the duration of a measure. A rest is a period of time in music when no sound is made. How many beats are in the rest?

**3.** 4/4 time means that there are 4 beats in a measure as indicated by the 4 on top. A quarter note gets one beat as indicated by the 4 on the bottom. Using this information, what does 3/4 time mean?

**4.** In 6/8 time, an eighth note gets one beat and there are 6 beats in a measure. A quarter note gets twice as many beats as an eighth note. How many beats does a quarter note get in 6/8 time?

**5.** A 16th note gets half as many beats as an eighth note. How many beats does a 16th note get in 6/8 time?

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# KEY: BELLE OF LOUISVILLE: HANDOUT 5

## Enrichment: Fractions in Musical Notation

The timing of music uses a special written notation. Written music begins with a time signature that has two numbers. Many songs are written in what is called 4/4 time or 3/4 time (pronounced “four four” and “three four”). The number on top (here 4 or 3) describes how many beats there are in a measure, the even time intervals into which written music is divided.

The bottom number describes which note receives one beat. In typical 4/4 time and 3/4 time, a quarter note gets one beat. The following notes are held for these durations with these time signatures.

Whole notes = 4 beats
1/2 notes = 2 beats
1/4 notes = 1 beat
1/8 notes = 1/2 beat
1/16 notes = 1/4 beat

**1.** For 3/4 time, choose a note or combination of notes that could complete a measure that starts with a half note.
*4 sixteenth notes*
*2 eighth notes*
*1 eighth note and 2 sixteenth notes*
*1 quarter note*

**2.** In one measure of music in 4/4 time, eight 16th notes are used with a rest for the duration of a measure. A rest is a period of time in music when no sound is made. How many beats are in the rest?
*2 beats*

**3.** 4/4 time means that there are 4 beats in a measure as indicated by the 4 on top. A quarter note gets one beat as indicated by the 4 on the bottom. Using this information, what does 3/4 time mean?
*There are 3 beats in a measure and the quarter note gets one beat.*

**4.** In 6/8 time, an eighth note gets one beat and there are 6 beats in a measure. A quarter note gets twice as many beats as an eighth note. How many beats does a quarter note get in 6/8 time?
*2 beats*

**5.** A 16th note gets half as many beats as an eighth note. How many beats does a 16th note get in 6/8 time?
*1/2 beat*

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