



Scale City: The Road to Proportional Reasoning is designed to provide middle school teachers and students with engaging and effective online tools for exploring the mathematics of scale and scaling. Although *Scale City* is easy to understand and navigate, the information contained in this guide may be useful to you as you decide how and when you want to incorporate the resources into your teaching.

The guide includes an introductory essay written by the project’s main content adviser, information about the various components of the *Scale City* web site, advice about different ways to use the resources in your classroom and about downloading the videos and interactives, eight “mystery” problems to use as class starters or exit activities, and a comprehensive assessment tool.

About the Project

Scale City was funded in part by a Local Services Initiative grant from the Corporation for Public Broadcasting. Kentucky Educational Television (KET) and three other state public television networks—Alabama Public Television, Arkansas Educational Television, and Maryland Public Television—joined together to form the CPB/LSI STEM (Science Technology Engineering Mathematics) Collaborative. All four partners were charged with developing online resources for middle grades mathematics that met a defined need in their respective states. You can link to our partners’ resources at “Other Attractions” on the *Scale City* web site. Like *Scale City*, those resources may be freely implemented in your classroom.

To help determine what topic would be most useful to Kentucky middle school teachers and students, KET convened a taskforce of teachers, university professors, Kentucky Department of Education staff, and other stakeholders in March, 2008. The project taskforce directed KET to develop online resources related to proportional reasoning, a topic that bridges arithmetic and higher mathematics and that is essential for later work in the sciences.

How We Went to *Scale City*

After the taskforce meeting, KET began to consider how to narrow the topic. To best utilize the visual nature of video and online Flash interactives and to provide students with real-life connections to proportional reasoning, KET decided to focus on the mathematics of scale and scaling. Research suggests that middle school students often have great difficulty understanding how to use proportionality to scale geometric figures or quantities and to explore their mathematical relationships. Choosing to focus on scale has the added advantage of giving students practice in measurement, another topic identified by the taskforce as crucial to student success and achievement.

Each *Scale City* module begins with a video visit to a place chosen for its local and universal appeal, from Dinosaur World’s kitschy collection of full-scale dino models in Cave City, Kentucky to the Louisville Slugger Museum and Factory, where the entrance is guarded by the world’s largest bat, to elaborate miniature collections in Maysville and Covington, Kentucky. These road stops provide inspiration for Flash interactive simulations that allow students to explore the mathematics of scale.



An Introduction to *Scale City*

Linda Sheffield, Ph.D.

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Dr. Sheffield is chair of the Math/Science Task Force of the National Association for Gifted Children (NAGC), working on initiatives in support of the identification and development of mathematical and scientific talent and promise in students of all ages. She has conducted seminars for teachers and students across the United States as well as in Japan, Singapore, England, Israel, Greece, the Czech Republic, Korea, Bulgaria, Mexico, India, Denmark, Spain, Germany, Sicily, Australia, China, and Hungary.

Proportional reasoning has been called both the capstone of elementary mathematics and the cornerstone of higher mathematics (Lesh, Post, Behr, 1988). Therefore, proportional reasoning is often considered one of the most important “big ideas” in middle school mathematics. Proportionality is also a thread that can tie together many of the major mathematical topics that are critical for developing a solid foundation for the mathematics that students will encounter in high school.

The National Council of Teachers of Mathematics (NCTM) in the seminal document, *Principles and Standards for School Mathematics*, noted, “Curricular focus and integration are also evident in the proposed emphasis on proportionality as an integrative theme in the middle-grades mathematics program. Facility with proportionality develops through work in many areas of the curriculum, including ratio and proportion, percent, similarity, scaling, linear equations, slope, relative-frequency histograms, and probability. The understanding of proportionality should also emerge through problem solving and reasoning, and it is important in connecting mathematical topics and in connecting mathematics and other domains such as science and art.” (NCTM, 2000) You will see many of these applications of proportional reasoning as you explore *Scale City*.

So what is **proportional reasoning** and why is it so important? **Proportional reasoning** involves the ability to make multiplicative comparisons among relationships. In elementary school, comparisons often involve addition and subtraction. These additive comparisons continue to be important, but as students move into intermediate grades and middle school, they should begin to use multiplicative reasoning (involving multiplication and division).

For example, if a student has a photograph that is 3 inches by 5 inches and wants to blow it up so that the shorter side is 6 inches, it is interesting to ask students how long the longer side should be. Some students will reason that since 6 inches is 3 inches longer than the original 3 inches, the longer side should now be 8 inches, which is 3 inches longer than the original 5 inches. However, this use of additive reasoning would not result in a photograph that is proportional to the original.

To find the dimensions of a larger photograph that is proportional to the original, students need to use multiplicative reasoning. The shorter side of the enlarged photograph is twice as long as the shorter side of the original, and therefore, the longer side must also be twice as long as the original, or 10 inches.



Proportional reasoning involves comparing two **ratios**, and each **ratio** is a comparison of two quantities. In the example above, you could think of one ratio as the comparison of the shorter side of the original to the shorter side of the enlarged photo or 3 inches to 6 inches or 3 to 6. This could also be written as $\frac{3}{6}$ or 3:6. The ratio of the longer side of the original to the longer side of the enlarged photo is 5 to 10 or $\frac{5}{10}$. You can find the **value** of each of these ratios by simplifying the fractions or by dividing the numerator by the denominator. Since the ratio $\frac{3}{6}$ and the ratio $\frac{5}{10}$ both have the same value of $\frac{1}{2}$ or 0.5, you know that these ratios form a **proportion** (which can be defined as two equivalent ratios), and the enlarged photo will look like the original, only larger.

One of the interesting things about ratios and proportions is that there are many ways of looking at them. In the photograph example, you could also look at the ratio of the shorter side to the longer side of the original photo (3 to 5) and compare it to the ratio of the shorter side to the longer side of the enlarged photo (6 to 10). Since both of these have a value of $\frac{3}{5}$ or 0.6, again you can conclude that these photographs are similar and the larger one is proportional to the smaller. Students might also wish to compare the ratios of longer sides to shorter ones (instead of shorter sides to longer ones). They should find a number of different equivalent ratios to demonstrate that the photos are proportional.

In geometry, **similar figures** are those that have the same shape, but do not necessarily have the same size. That means that congruent figures are a special type of similar figures. For all similar two-dimensional figures, their sides are proportional; and for all similar three-dimensional figures, their edges are proportional. A **scale factor** is the ratio of corresponding lengths of the sides or edges of two similar figures.

In the example above, a scale factor of two was used to enlarge the original photo. The length of each side of the enlarged photo is twice as long as the length of the corresponding side of the original photo. If a scale factor of $\frac{1}{3}$ had been used, the length of each side of the new photo would be $\frac{1}{3}$ as long as the length of the original photo and the new photo would be much smaller. If a scale factor of 1 were used, the new photo would be congruent to the original. Students will explore this one-dimensional scaling of proportional figures as they make a diorama of dinosaurs and find the heights of objects such as the Louisville Slugger bat using shadows. These same scaling ideas and proportions will also be used as they scale up recipes.

In *Scale City*, students will also explore what happens to the area of a two-dimensional figure like a mural and what happens to the surface area and volume of three-dimensional scale models when different scale factors are used. They will note that although they can find the length of each side of the new similar two-dimensional figure by multiplying each corresponding side of the original by the scale factor, to find the area of the new figure, they must multiply the area of the original figure by the square of the scale factor. This will also be true when finding the surface area of similar three-dimensional figures. In finding the volume of similar three-dimensional figures, students should discover that they could multiply the volume of the original figure by the cube of the scale factor to find the volume of the new figure. All of this will be done in a hands-on manner, manipulating interactives and/or concrete objects to develop a deeper understanding of these concepts.

So far, this discussion has been about **direct relationships**. In a **direct relationship**, when one quantity gets larger, a second quantity gets larger at the same time. When this relationship is a **direct proportion**, it can be said that when one quantity changes, a second quantity changes by the same factor. The lengths of the sides of similar figures are a **direct proportion**. If x is the length of one side of a figure and y is the length of the corresponding side of a similar figure, we might say that $x = k \cdot y$ where k is the scale factor, which is also called the **constant of variation**. This same k (constant of variation or scale factor) will be the ratio between any two corresponding sides of similar figures.

Sometimes the relationship between two quantities is an **inverse relationship**. In an **inverse relationship**, when one quantity gets larger, the second quantity gets smaller. If this is an **inverse proportion**, when you multiply the two quantities together, you will get a constant. This is sometimes written as $x \cdot y = k$, where k is the constant of variation. Students will explore inverse proportions at the drive-in



theater as they measure what happens to a shadow as a person gets closer or farther from a light source, and in music as they explore what happens to the pitch of a note as a string or pipe gets longer or shorter. When they visit the Kentucky Horse Park, they will explore both direct and inverse proportions as they investigate problems using speed, time and distance.

Tables and graphs are a great way to help students see both direct and inverse relationships. In several of these interactives, students will record data in a table and graph the results on a Cartesian coordinate system. They will find that when they plot points representing a direct proportion, the points all lie on the same line (which passes through the origin.) This is another way of saying that all direct proportional relationships are **linear relationships**. The slope of the line is actually the constant of variation, although that is not explored in depth in these interactives. When the points from a table showing an inverse proportion are graphed, the points no longer lie on a line. Instead, these points form a curve that never touches the x -axis or the y -axis.

Throughout *Scale City*, the emphasis is on helping students make sense of proportionality. Research has shown that too often the emphasis in middle grades mathematics has been on defining a proportion as two equivalent ratios and then rotely teaching students to use **cross-multiplication** to solve for the unknown value in the proportion. The NCTM *Principles and Standards for School Mathematics* states, “Instruction in solving proportions should include methods that have a strong intuitive basis. The so-called cross-multiplication method can be developed meaningfully if it arises naturally in students’ work, but it can also have unfortunate side effects when students do not adequately understand when the method is appropriate to use. Other approaches to solving proportions are often more intuitive and also quite powerful.” (NCTM, 2000, p. 220) We hope that as students take this trip through *Scale City*, they will not only enjoy the journey, but will also become competent, confident, and powerful in their use of proportional reasoning.

References:

Lesh, R., Post, T., and Behr, M. (1988). Proportional reasoning. In J. Hiebert and M. Behr (Eds.), *Number Concepts and Operations in the Middle Grades* (pp. 93 – 118). Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.



About the Learning Modules

Scale City: The Road to Proportional Reasoning is organized into eight learning modules. These modules, separately and collectively, are designed to help students explore and understand concepts related to proportional reasoning and to scale and scaling in particular.

Here is a list of the modules and their main topics, arranged from the simplest concepts to the most complex.

- **Greetings from Dinosaur World**
One-dimensional scaling: Using proportions and the known height of an object to find an unknown height
- **Greetings from the Louisville Slugger Museum**
One-dimensional scaling: Using similar triangles and shadows to find an unknown height
- **Greetings from the World of Mural Painting**
Two-dimensional scaling: Seeing what happens to area when the dimensions of murals are increased proportionally
- **Greetings from Miniature Land**
Three-dimensional scaling: Increasing the dimensions of a house by a factor of 10
- **Greetings from the World Chicken Festival**
Scaling up recipes and a skillet to cook for a crowd
- **Greetings from Sky-View Drive-In**
Discovering the inversely proportional relationship between the size of an object's shadow and its distance from a light source
- **Greetings from the Belle of Louisville**
Exploring the inversely proportional relationship between the length of musical pipes or strings and their pitch or frequency
- **Greetings from the Kentucky Horse Park**
Learning about the directly and inversely proportional relationships among time, distance, and speed

Web Components for Learning Modules

The web page for each *Scale City* destination includes a three- to six-minute video fieldtrip, a related interactive simulation, and a guide to the interactive. Accompanying lesson plans, assessments, handouts, and keys are posted at the Teacher's Diner.

Additional web components include the Scale City Quiz, which you can use for review or assessment, and Other Attractions, where you can link to the web resources created for middle grades math students and teachers by KET's public television partners.



Videos

Eight short videos take students to interesting places related to the mathematics of scale. Their purpose is to showcase real-life applications of proportional reasoning and to introduce a mathematical concept that students can explore in the interactive simulations, lessons, and handouts. They also provide connections to other subjects, from science and history to music and art.

Bandwidth Alternatives

Videos are offered in three sizes—small, medium, and large. These designations refer to the bandwidth required to view them. Most teachers will probably prefer the medium or large size for whole class viewing, but all three can be projected full-screen or viewed in a smaller window on individual computers. If you are using a CD-ROM version of the web site, rather than watching the videos online, only one video size is included, since bandwidth is not an issue.

Closed Captioning

If you want, you can click “cc” on the video player to activate closed captioning. This feature is designed for students with hearing impairments, but it also can be useful if you don’t have a good classroom speaker system or if you want students to turn down the audio while watching the videos individually.

Special Resources

Two of the videos are accompanied by additional PDF resources—a playful, yet educational guide to the dinosaurs appearing in the “Greetings from Dinosaur World” video and an artist’s statement from the painter whose murals are featured in the video and interactive for “Greetings from the World of Mural Painting.” The latter includes information about how the artist uses math to create murals and images of his murals.

Interactive Simulations

Each video is paired with a Flash simulation that allows students to learn about a mathematical concept through interactive features and problems. The simulations are designed to be fun, engaging, and easy to manipulate. Most include “Help” buttons that offer additional advice and information about using the interactive.

Flexibility of Design

Although they were produced in conjunction with the videos, the interactives also may be used independently to help teach, review, or reinforce a concept. They can be used with a projector or Smart Board for whole class instruction, in small groups, or individually in a computer lab, home, or library.

Zooming In

Mac users can use control click and PC users can right-click to zoom in on any feature of the interactives. The more you zoom in, the more detail you will see. This feature is particularly useful for zooming in on the Din-O-Rama interactive and for getting a closer look at the fun “snacks” you can win at the Scale City Quiz.

Using Calculators To Solve Problems Presented in Interactives

A number of math problems included in the simulations are simple computations that most students can solve using mental math or estimation. Others will require a calculator or pencil and paper calculations. During whole-class instruction, you can use your computer’s calculator app to solve these problems or have your students use hand-held calculators at their desks.

Guides to Interactives (Including Printable Forms)

Guides to the interactives (PDFs) are linked from each module web page. These guides include hints and advice for manipulating the simulations, as well as write-ups of all questions and problems.



You can distribute these forms as part of whole class instruction so students can solve the problems at their desks as they watch the simulations on a classroom projector. Or you can have students complete the forms for homework or class work while they use the interactives individually or in small groups. Since the answers to the interactives can't be saved electronically, this feature gives you a way to evaluate individual student effort.

Teacher's Diner

At the Teacher's Diner, you will find a resource list, a vocabulary list, and lesson plans and handouts for each of the eight modules, all in PDF format.

Resources

This annotated list includes interactive online resources; computer software; classroom materials and manipulatives; books for teachers, picture books, fiction, and films; and resources related to individual learning modules. All resources include information and/or tools to help teach proportional reasoning.

Vocabulary

The vocabulary list provides definitions of terms used throughout the modules, along with applications drawn from the lessons and interactives. This list can be used as a reference tool for teachers or students.

Lesson Plans

Scale City lesson plans are designed for flexibility. If you want, you can follow them step-by-step as guides for instruction in your classroom. Or you can use one or more elements (i.e., an activity, an assessment, a review, etc.) to supplement or extend existing lessons.

In addition to instructions, ideas, and activities for a multi-day lesson built around the videos and Flash interactives, the lesson plans include:

- Alignment to Kentucky education standards, which are based on national mathematics education standards
- Sample e-mails to send to parents
- Writing prompts
- Adaptations for diverse learners and lesson extensions
- Ideas for cross-curricular connections to share with colleagues in other disciplines
- Links to online resources related to the lesson's subject
- Assessments (multiple choice, performance, and open response) with keys incorporated into the lesson plan

Handouts

Every lesson plan is supplemented by five to 10 handouts—viewing guides for video field trips, word problems related to lesson content, questions to use in classroom games and activities, and more. Like the lessons, the handouts are designed for flexibility—you can use them exactly as suggested in the lesson plans or incorporate them into your own instructional plans. For your convenience, a key accompanies each handout.

Teaching Tips

Scattered throughout the lessons and even in some of the handout keys are special notes to the teacher and teaching tips. These teaching tips and notes discuss teaching techniques and approaches, provide ideas for extending the lessons, and offer additional explanations of terms or difficult concepts.



Scale City Quiz

For review, pre- or post-assessment, or just for fun, the Scale City Quiz uses an old-fashioned vending machine and a game-like structure to test students' knowledge of the proportional reasoning concepts taught in the eight interactive simulations. Unlike the module interactives, the Quiz is primarily designed for individual use.

Players have to answer two questions based on each interactive. When they get an answer right, a green button lights up on the left side of the machine. When they get one wrong, a red button lights up and a hint appears below the machine.

When players get both answers right, a snack drops down. If they get too many wrong answers, they run out of quarters and have to start over. The object is to win all eight snacks.

When you mouse over images from the interactives that appear on the screen beside questions, they automatically enlarge so you can see their details more clearly. Be sure to zoom in on the snacks in this interactive—their tongue-in-cheek designs will appeal to students and adults alike.

Other Attractions

“Other Attractions” briefly describes the projects developed by KET’s public television partners—Alabama Public Television, Arkansas Educational Television, and Maryland Public Television—and provides links to their web sites. There you’ll find free resources including both online interactives and videos as well as related teacher materials.

As a bonus, the Other Attractions page provides a PDF of “World’s Largest ...” sites in Alabama, Arkansas, and Maryland and a link to a “World’s Largest...” web site where you can explore scale-related locations in other states.



Alternatives for Using *Scale City*

Classroom Setup

Whole Class

Both the videos and the Flash simulations can be used whole class with a Smart Board or other projection device. Each size alternative for the videos—small, medium, and large—can be shown full-screen for whole-class viewing. Using the medium or large version will result in a clearer full-screen image.

You also can show the Flash simulations full screen. The font sizes in these simulations are large enough to be read at a distance, and students can use the forms provided in the interactive guides to fill in answers to questions and problems.

Alternatively, if you have a classroom response system or “clickers,” you could use these devices to keep students engaged in the interactives and to assess their understanding. Students could also take turns coming up to the front of the class and answering the questions posed on the interactives.

Small Group

If you have several classroom computers with Internet connections, you may want to have groups of students work on the interactives together. Again, they could use the forms provided in the guides to record their answers to the questions and problems. This configuration would allow students of varying abilities to help one another.

Individual Work

Students can work on the interactives on their own, in computer labs, in the library, or at home. You might encourage them to visit the web site with their families to reinforce what they have learned in class. If you want, you could require them to use the forms provided in the interactive guides to document their work and their answers.

Unlike the other interactives, the Scale City Quiz is designed for individual use, although it also could be used in pairs or small groups.

Instructional Alternatives

There’s no right or wrong way to incorporate *Scale City* resources into your classroom. You might decide just to use one or two of the videos and/or the interactives to introduce or review a concept. Along with the video and/or interactive, you could use all or most of the accompanying lesson and handouts. Alternatively, you could develop your own activities and lessons around the resources. Many of handouts and assessment activities themselves could be integrated into your existing curriculum, with or without the videos and interactives.

If you do decide to use just one or two of the learning modules, you may need to review ratio, proportion, and scale with your students before you begin. These concepts are introduced in the Dinosaur World and Louisville Slugger Museum modules, and the later lessons assume that students already have been introduced to them.



Using Scale City as an Extended Unit

In its entirety, *Scale City* provides several weeks of instruction on proportional reasoning, ratios, proportion, and scale. By following the “Suggested Route” on the home page, you and your students will start with the easiest concepts and move on to more difficult ones.

You will find that the individual lessons offer options for shortening or extending the number of days you spend on a topic. There is easily enough material for an eight- or nine-week special, but you could shorten the time you spend on the lessons to three or four weeks. It’s up to you.

For your convenience, Appendix 2 of this guide includes a comprehensive multiple-choice assessment that could be used at the end of an extended unit based on *Scale City*.

Problems Playing and Downloading the Videos and Interactives

Scale City requires Adobe Flash Player 10.0.12.36 or higher. If you can’t play the video or you have problems with the interactives not working properly, your Adobe Flash Player may need to be updated. Visit the Adobe web site at get.adobe.com/flashplayer/ to download the latest free Flash Player plug-in.

If you are teaching in a computer lab setting, with multiple computers trying to download the same interactive simultaneously, you may find that the interactive loads very slowly. “Mural Math” (from “The World of Mural Painting”) and “House of Scales” (from “Miniature Land”) are the largest files, so they will load the slowest.

To deal with this problem, you might have students immediately begin downloading the interactive when they get to the lab so that they can do other activities while the interactive is loading. Once the interactives have been downloaded once, they most likely will be cached on the computer, so they will download more quickly next time.



Appendix One: Scale City Ticklers

The ticklers in this appendix are everyday mysteries related to the concepts taught in *Scale City*. They give students a simple mathematical mystery to solve that introduces or reviews concepts related to each *Scale City* module.

The ticklers appear on separate pages, so you can use them as handouts if you like. Answers are provided at the end of this appendix.



DINOSAUR WORLD

The Mystery of the Unknown Heights

Name:

Date:

The senior class went to a natural history museum. Later, the yearbook staff was looking at pictures of a group of students beside a life-size model of a small dinosaur. The basketball manager said he knew the height of one of the students—Ben, a basketball player who is 6 feet tall. Then he said he could use that information and the picture to figure out the heights of all the students and of the dinosaur.

When the students checked, they found that the manager had correctly identified the heights. How did he do it?



LOUISVILLE SLUGGER MUSEUM

The Mystery of the Invisible Thieves

Name:

Date:

A statue has been stolen from the city's history museum. The theft took place on Monday, the day that the museum is closed. The thieves apparently knew where the outside security camera was located, because they avoided getting their picture taken. However, the security camera did capture a clear image of the shadows of two people carrying a statue beside the dumpster and its shadow. How could a police detective use this photograph to determine the heights of the thieves?



MURAL PAINTING

The Mystery of the Bigger Painting

Name:

Date:

Melanie was asked to recreate a larger version of a landscape painting she calls “Solitude” that features many shades of blue and green. For a 17 ½-inch by 22-inch canvas, she used 6 ounces of blue paint and 4 ounces of green paint. The teacher has asked her to re-create the piece on a 35-inch by 44-inch canvas. Melanie says she knows exactly how much blue and green she needs. How could she know this before painting the larger version?



MINIATURE LAND

The Mystery of the Scale Drawings

Name:

Date:

The architecture club is making drawings of great buildings and structures of the world. The club advisor has told them to use an architectural scale in which 1/16 inch in the drawing represents one foot in the actual structure. When Colin brought in drawings of the Statue of Liberty and the Washington Monument, the club advisor knew instantly that he was not using the scale correctly. The Statue of Liberty's torch came to the top of the Washington Monument in Colin's drawings. How did the advisor know so quickly that Colin had had a problem with scale?



WORLD CHICKEN FESTIVAL

The Mystery of Granny's Safe

Name:

Date:

Great-Great Granny Burkes had a funny sense of humor. In her will, she left her great-great grandchildren a biscuit recipe, a safe, and this clue to the combination.

This secret biscuit recipe made two dozen or so.
Plan for 72 and you might get the dough.
Take note of three ingredients. The smallest mean a lot.
Turn left, right, left. And you'll get the money pot.

Granny Burke's Biscuits

5 cups flour
7 teaspoons baking powder
1 teaspoon cream of tartar
1/3 teaspoon salt
1 cup butter
1 1/2 cups milk



SKY-VUE DRIVE-IN

The Mystery of the False Puppeteer

Name:

Date:

At the end of the day, the substitute teacher reported an incident that occurred while students were watching a film. She didn't remember any of the students' names or who was making trouble. She only knew the disruptive student was a boy with blonde hair. Four boys fit the description, so the vice-principal interviewed all four the next day.

Three boys said similar things during their interviews. "The other boy caused a lot of problems in class yesterday," they said. "His seat is in the front of the class close to the screen. We could see him from where we were sitting in back. He was making all these shadow puppets, so we couldn't see the movie at all. We could only see his shadow puppets covering the entire screen."

The fourth boy had a different story. "I didn't make shadow puppets. My seat is in the front of the room. The shadow images were made by the boy sitting next to the projector."

You might think that three testimonies were stronger than one, but not in this case. The vice-principal sent the boy with the different story back to class. Why did he think the other boys were not telling the truth?



BELLE OF LOUISVILLE

The Mystery of the Wrong Wind Chimes

Name:

Date:

As a retirement gift for the music teacher, the shop class made wind chimes. The president of the music booster club heard about the wind chimes, and requested another set an octave lower using the same kind of pipe. She offered to pay for the materials. After the wind chime was ready, she called the shop teacher to find out how much she owed him.

The shop teacher said this set of wind chimes would be cheaper than the original set, because the class had used less pipe. The booster club president knew that the wind chimes had not been made correctly, even though she hadn't seen or heard them. What tipped her off?



KENTUCKY HORSE PARK

The Mystery of the Speeding Sister

Name:

Date:

Emily's older sister, Anna, was grounded for speeding on her way back home from visiting a college. Anna did not receive a speeding ticket, and no one reported speeding to her parents. Still Emily's mother grounded Anna when she walked in the door.

The college Anna was visiting was 140 miles from home. Anna called before leaving at 5:20 pm and arrived home at 7:12 pm. The speed limit on the road Anna was driving is 65 mph. How did her mother know Anna had been speeding?



SCALE CITY TICKLERS

KEY

Dinosaur World: The Mystery of the Unknown Heights

The basketball manager used proportional reasoning to find the height of the students and the dinosaur. He knew Ben's height, since Ben played for the team. So the manager used the picture, a ruler, and a calculator. Knowing one height and measuring the height of that person in the picture gave him the ratio of the picture's size to the actual size of the people and the dinosaur.

For example, if Ben were 3 inches tall in the photograph, the ratio of the picture's size to actual size, would be 3 inches to 6 feet, or 1 inch to 2 feet. Another way of representing the scale would be 1 inch to 24 inches or 1:24.

On this scale, if George's height in the photograph is $2\frac{3}{4}$ inches, his real height is 5 feet 6 inches. If Kara's height in the photograph is $2\frac{5}{8}$ inches, her real height is 5 feet 3 inches. And if the dinosaur figure is $4\frac{1}{2}$ inches tall, its real height is 9 feet.

Louisville Slugger Museum: The Mystery of the Invisible Thieves

To determine the height of the thieves, the detective needs to use proportional reasoning in two different ways. First, she needs to determine the scale of the photograph. She can do this by measuring the actual height of the dumpster and the height of the dumpster in the photograph. If the real dumpster is 9 feet tall, and its image in the photo is 3 inches tall, the scale is 3 inches to 9 feet, or 1 inch to 3 feet. Another way of expressing this ratio is 1 inch to 36 inches.

Then the detective should measure the shadow of the dumpster in the photograph and the shadows of the two thieves. The triangle formed by the dumpster and its shadow is similar to the triangle formed by the thieves and their shadows, so the value of the ratio of the dumpster's height to the its shadow's length will be equal to the value of the ratio of each thief's height to his shadow's length.

Dumpster's height/Dumpster's shadow's length = Thief's height (unknown)/Thief's shadow's length

Let's say the shadow of the dumpster is 4 inches long. So the value of the ratio of the dumpster's height to its shadow's length is $3 \div 4$, or 0.75. If one of the thieves' shadows is $2\frac{1}{2}$ inches long, his height in the scale of the picture would be 2.5×0.75 or 1.875 inches ($1\frac{7}{8}$ inches). In the scale of the photograph, that's equal to 5 feet $7\frac{1}{2}$ inches. If the other thief's shadow is $2\frac{5}{8}$ inches long, his height in the scale of the photograph would be $2\frac{5}{8} \times \frac{3}{4}$ (the fractional equivalent of 0.75) or $\frac{63}{32}$ or $1\frac{31}{32}$ inches. Multiply this fraction by 36 to get the actual height, and you get $\frac{2,268}{32}$, which equals 70.875 or 6 feet and $\frac{7}{8}$ inches. So that's the height of the taller thief.

Mural Painting: The Mystery of the Bigger Painting

Melanie used mathematics to determine how much paint she needed for the larger painting. The bigger painting is twice as long and twice as wide as the smaller painting, so it will have four times as much area and use four times as much paint. She will need 24 ounces of blue and 16 ounces of green to recreate the piece on the 35-inch by 44-inch canvas.



KEY APPENDIX ONE: SCALE CITY TICKLERS

Miniature Land: The Mystery of the Scale Drawings

The advisor must be familiar with the heights of the Washington Monument and the Statue of Liberty. Since the Washington Monument is 250 feet taller than the Statue of Liberty, Colin's scale drawings could not be close to the same height if Colin drew them on the same scale.

Structure	Actual Height	Height in Inches	Height in 1/16 Inch to 1 Foot Scale
Washington Monument	555 feet 5 1/8 inches	6665 1/8 inches or 6665.125	34.71 inches
Statue of Liberty	305 feet 1 inch	3661 inches	19.07 inches

World Chicken Festival: The Mystery of Granny's Safe

The ingredients with the smallest amounts are baking powder, cream of tartar, and salt. Even though the amounts are small, these ingredients "mean a lot" because they determine the way the biscuits rise.

Two dozen equals 24 biscuits. If you make for 72, you have to triple the recipe.

To triple the recipe, you need to multiply each ingredient by 3. The tripled amounts are 21 teaspoons baking powder, 3 teaspoons cream of tartar, and 1 teaspoon salt. So the safe's combination is left 21, right 3, and left 1.

Sky-Vue Drive-In: The Mystery of the False Puppeteer

The shadow puppets would be the biggest when the hands creating the puppets are closest to the projector. The three boys must have discussed their story. They described a situation that is not possible.

Belle of Louisville: The Mystery of the Wrong Wind Chimes

The president of the music booster club requested that the chimes be made using the same kind of pipes. Chimes an octave lower would require pipes twice as long, not pipes that are shorter than the original ones.

Kentucky Horse Park: The Mystery of the Speeding Sister

To drive 140 miles in 1 hour 52 minutes, Anna must have driven an average of 75 mph, which is greater than the speed limit. Even with a 70-mph speed limit, Anna could not have traveled the distance in less than 2 hours without speeding. With a 65-mph speed limit, particularly considering that she must have slowed down when she got off the interstate to go to her house, Anna could not have traveled the distance in less than 2 hours 9 minutes without speeding.



Appendix Two: *Scale City* Unit Assessment

If you use *Scale City* as an extended unit, these multiple-choice questions can be used as a final assessment for your students. They also could be used for review.

Key to Unit Assessment

1. A, 2. D, 3. D, 4. A, 5. B, 6. C, 7. A, 8. B, 9. A, 10. B, 11. C, 12. D, 13. B, 14. A, 15. D, 16. D, 17. C, 18. C, 19. D, 20. A, 21. C, 22. A, 23. C, 24. B, 25. D



MULTIPLE CHOICE ASSESSMENT

Scale City Unit Assessment

Name:

Date:

1. Harley is 33.33% of a dinosaur's height. If Harley is 5 feet tall, the dinosaur would be
 - A. 15 feet tall.
 - B. 12 feet tall.
 - C. 10 feet tall.
 - D. 7.5 feet tall.

2. The relationship of shadow height to distance from the projector is an inverse proportion. When Lu was 12 feet from the projector, her shadow was 14 feet tall. When Lu stands 24 feet from the projector, her shadow will be
 - A. 28 feet tall
 - B. 14 feet tall
 - C. 12 feet tall
 - D. 7 feet tall

3. There are 8 hot dogs in a pack and 12 buns in a bag of hot dog buns. Chip needs to buy enough for 108 hot dogs. He should buy
 - A. 10 packs of hot dogs and 6 bags of hot dog buns
 - B. 9 packs of hot dogs and 14 bags of hot dog buns
 - C. 12 twelve packs of hot dogs and 8 bags of hot dog buns feet tall
 - D. 14 packs of hot dogs and 9 bags of hot dog buns

4. Speed is
 - A. distance \div time
 - B. distance \cdot time
 - C. time \div distance
 - D. distance plus time

5. If the diameter of a circle is 4.3 feet, the circumference of the circle is approximately
 - A. 14.5 feet
 - B. 13.5 feet
 - C. 3.1 feet
 - D. 1.4 feet



APPENDIX TWO: MULTIPLE CHOICE UNIT ASSESSMENT

- 6.** The recipe requires $\frac{3}{4}$ teaspoon Worcestershire sauce. The recipe will be scaled up for the party with each ingredient multiplied by 8. Since three teaspoons make a tablespoon, the amount of Worcestershire sauce needed is
- A. 3 teaspoons
 - B. 1 tablespoon
 - C. 2 tablespoons
 - D. 3 tablespoons

- 7.** A gallon of paint will cover 400 square feet. Jeb is painting the floors of four classrooms, each of which measures 20 feet by 30 feet. He should buy
- A. six gallons of paint
 - B. four gallons of paint
 - C. three gallons of paint
 - D. two gallons of paint

- 8.** Each student was given 12 building blocks of the same size and told to build a rectangular structure. The x and y represent the dimensions of the sides.

x	12	6	4	3	2
y	1	2	3		

The unknown values for y would be

- A. 4 and 5
 - B. 4 and 6
 - C. 6 and 8
 - D. 24 and 36
- 9.** The exhibit of miniatures is on a 1:12 scale. A figure made to represent a six-foot man would be
- A. 6 inches tall
 - B. 8 inches tall
 - C. 12 inches tall
 - D. 16 inches tall
- 10.** The shadow of a 45-foot tall building is 60 feet long. At the same time of day, the 9-foot street sign would have a shadow of
- A. 16 feet
 - B. 12 feet
 - C. 10 feet
 - D. 9 feet



APPENDIX TWO: MULTIPLE CHOICE UNIT ASSESSMENT

- 11.** The history club is putting together a time capsule. One metal box measures 3 inches by 5 inches by 2 inches. Another metal box measures 6 inches by 10 inches by 4 inches. The volume of the larger box is
- A. twice the volume of the smaller box
 - B. three times the volume of the smaller box
 - C. four times the volume of the smaller box
 - D. eight times the volume of the smaller box
- 12.** There are about 3.1 miles in a 5-kilometer race. Finn Mott finished the 5K race in 18 minutes. His speed in mph is closest to
- A. 3.6 mph
 - B. 5.8 mph
 - C. 6.4 mph
 - D. 9.3 mph
- 13.** The speed limit was 40 mph in the 20-mile long construction zone. The rest of the trip, the family averaged 65 mph. If they traveled 150 miles total, their travel time would be
- A. $\frac{1}{3}$ hours
 - B. $2\frac{1}{2}$ hours
 - C. $2\frac{1}{4}$ hours
 - D. 2 hours
- 14.** In an inverse proportion, as one variable increases
- A. another variable decreases proportionally
 - B. there is no effect on the other variable
 - C. another variable increases proportionally
 - D. another variable equals zero
- 15.** The relationship between the lengths of pan pipes and their frequency is an inverse proportion. The pipe for C4 is 8 inches long. The pipe for C5 is 4 inches long. The frequency for C4 is 261.6 Hz. The frequency for C5 is
- A. 130.8 Hz
 - B. 261.6 Hz
 - C. 392.4 Hz
 - D. 523.2 Hz
- 16.** The architect's model building is 9 inches tall. Each foot of the actual building is represented in the model by $\frac{1}{16}$ of an inch. The actual building would be
- A. 36 feet tall
 - B. 72 feet tall
 - C. 128 feet tall
 - D. 144 feet tall



APPENDIX TWO: MULTIPLE CHOICE UNIT ASSESSMENT

17. Consider the table below.

x	y	k
2	32	64
4	16	64
8	8	64
16		64

When x is 16, y should be

- A. 6
- B. 5
- C. 4
- D. 2

18. The string was 16 inches. A string made from the same material that produces a sound twice the frequency of the 16-inch string would likely be

- A. 32 inches long
- B. 16 inches long
- C. 8 inches long
- D. 4 inches long

19. The flagpole outside the school is 20 feet tall. Its shadow is 10 feet long. If John is 6 feet tall, his shadow at the same time of day would be

- A. 12 feet long
- B. 6 feet long
- C. 4 feet long
- D. 3 feet long

20. If the frequency of a note is 220 Hz, a note an octave lower should have a frequency of

- A. 110 Hz
- B. 220 Hz
- C. 440 Hz
- D. 660 Hz

21. One triangle has sides measuring 3 cm, 4 cm, and 5 cm. A larger triangle similar to this triangle has a side measuring 12 cm. The other sides could measure

- A. 6 cm and 8 cm
- B. 16 cm and 18 cm
- C. 9 cm and 15 cm
- D. 9 cm and 18 cm



APPENDIX TWO: MULTIPLE CHOICE UNIT ASSESSMENT

- 22.** In a picture from Dinosaur World, Kelsey is 3 inches tall and her younger sister is $2\frac{1}{2}$ inches tall. Kelsey is 5 feet, 6 inches tall. Her sister is closest to
- A. 4 feet, 6 inches tall
 - B. 3 feet, 10 inches tall
 - C. 5 feet, 2 inches tall
 - D. 5 feet, 4 inches tall
- 23.** The artist used 2 ounces of green on an $8\frac{1}{2}$ -inch by 11-inch paper. To recreate the same work on 17-inch by 22-inch paper, the amount of green paint needed would be
- A. 4 ounces.
 - B. 6 ounces
 - C. 8 ounces.
 - D. 12 ounces.
- 24.** If you compare a rectangle that is 4 feet by 8 feet with one that is 12 feet by 24 feet, the smaller one has
- A. has one-third as much area as the bigger one
 - B. has one-fourth as much area as the bigger one
 - C. has one-ninth as much area as the bigger one
 - D. has one-sixteenth as much area as the bigger one
- 25.** Trevor's sister is $\frac{1}{8}$ the height of a 40-foot dinosaur. She is
- A. 6 feet tall.
 - B. 5.5 feet tall
 - C. 5.3 feet tall
 - D. 5 feet tall

